ARS 42-79
November 1963

UNITED STATES DEPARTMENT OF AGRICULTURE
Agricultural Research Service

AN APPLICATION OF DATA SMOOTHING BY AN ANALOG COMPUTER

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Filtering by means of an analog computer to separate an unwanted random signal from a lower frequency signal is discussed. The spectral concept of the filtering problem is mentioned and an example of an analog computer filter circuit is given.

INTRODUCTION

The smoothing of electrical signal data by filtering on an analog computer is not a particularly new procedure. There are, of course, more sophisticated analog methods of extracting a desired signal which is intermingled with other signals. However, we shall briefly discuss the subject from the standpoint of filtering to separate an unwanted random signal from a more slowly fluctuating signal. We hope that this paper will have added benefit for the person who has not previously encountered references on the general technique.

The filtering method presented was used by the second author for smoothing the output signal from a force transducer in coefficient of friction measurements for agricultural product materials and by the first author to treat densitometer signals in the determination of background densities on spectrographic films.

The reader will find the references by Korn and Korn [2] and Jackson [3] helpful in regard to the simulation procedures used.

GENERAL THEORY

In the case under discussion it is desired that the output signal should have a narrow, low-frequency spectral density function $\Psi(\omega)$. For an input signal spectral density $\Phi(\omega)$, the linear filter of characteristic $\Theta(\omega)$ will operate on $\Phi(\omega)$ in the manner

$$\Psi(\omega)=\Theta(\omega)\Phi(\omega). \tag{1}$$

We now consider the filter analogous to the system

$$\dot{V}(t)+\alpha V(t)=U(t). \tag{2}$$

[1] Pioneering Research Laboratory on Physics of Fine Particles, Agricultural Engineering Research Divisions, ARS, USDA, and the Department of Agricultural Engineering, Ohio Agricultural Experiment Station, Wooster, Ohio, respectively.

[2] G. A. Korn and T. M. Korn. *Electronic Analog Computers.* 2d. ed. New York. 1956.

[3] A. S. Jackson. *Analog Computation.* New York. 1960.

In the manner of Batchelor,[4] we write

$$V(t)=\int_{-\infty}^{\infty} e^{it\omega}dZ(\omega)\,;\; U(t)=\int_{-\infty}^{\infty} e^{it\omega}dW(\omega), \tag{3}$$

and define

$$\lim_{d\omega\to 0}\frac{\overline{dZ(\omega_1)dZ^*(\omega_2)}}{d\omega}=0;\quad \lim_{d\omega\to 0}\frac{\overline{dW(\omega_1)dW^*(\omega_2)}}{d\omega}=0,\quad (\omega_1\neq\omega_2) \tag{4a}$$

$$\Psi(\omega)=\lim_{d\omega\to 0}\frac{\overline{dZ(\omega)dZ^*(\omega)}}{d\omega};\quad \Phi(\omega)=\lim_{d\omega\to 0}\frac{\overline{dW(\omega)dW^*(\omega)}}{d\omega}. \tag{4b}$$

Equations (3), (4a), and (4b), when applied to (2), produce the result

$$(i\omega+\lambda)dZ(\omega)=dW(\omega), \tag{5}$$

and further,

$$(\omega^2+\lambda^2)dZ(\omega)dZ^*(\omega)=dW(\omega)dW^*(\omega). \tag{6}$$

Hence, we finally obtain

$$\Psi(\omega)=\frac{1}{\omega^2+\lambda^2}\Phi(\omega). \tag{7}$$

Comparison of (1) and (7) shows that

$$\Theta(\omega)=\frac{1}{\omega^2+\lambda^2} \tag{8}$$

represents a low-pass filter which will give a $\Psi(\omega)$ of the desired type.

THE FILTER CIRCUIT

In analogy, suppose we wish to examine the velocity of a sphere of mass m subjected to an arbitrary force $u(t)$ and immersed in a viscous fluid. We presume that Stokes' law dissipation exists, i.e., the viscous force F_v will be given by

$$F_v=-\mathrm{K}(dx/dt). \tag{9}$$

If random fluctuations are superimposed on $u(t)$, it will be obviously difficult to measure the velocity. However, if $u(t)$ remains unchanged, and we place the sphere in a more viscous fluid, the random fluctuations in the velocity will be more heavily damped in accordance with the differential equation.

$$\frac{d^2x}{dt^2}+\frac{\mathrm{K}}{m}\frac{dx}{dt}=\frac{1}{m}u(t). \tag{10}$$

For the velocity $v=dx/dt$, (10) becomes

$$\frac{dv}{dt}+\frac{\mathrm{K}}{m}v=\frac{1}{m}u(t). \tag{11}$$

[4] G. K. Batchelor. *The Theory of Homogeneous Turbulence.* London. 1953.

We note that (11) is of the same form as (2). Therefore, placement of the sphere in the more viscous fluid would be analogous to the use of a low-pass filter. Taking the Laplace transform of (11) and assuming initial conditions are zero, we find that

$$V(s)=\left[\frac{1}{K\left(\frac{ms}{K}+1\right)}\right]U(s). \tag{12}$$

The first factor on the right-hand side of (12) is the type of transfer function needed to simulate this low-pass filter. Figure 1 shows the appropriate operational amplifier connection, for which the output X_0 in terms of the input X_1 is [5]

$$X_0=-\frac{R_0}{R(R_0Cs+1)}X_1. \tag{13}$$

With R_0 constant, the capacitance C is directly proportional to the time constant of the circuit, which in turn governs the frequency-cutoff point of the filter.

The circuit in figure 1 does not have the sharp cutoff characteristics of some more complex filters. However, it was very adequate for this work and there was good recorder-driving capability.

APPENDIX

An example of the use of the filter circuit in Figure 1 was the separation of unwanted higher frequencies from the desired low frequency signal of a friction coefficient force transducer. The circuit was implemented on a Boeing Model 6631 analog computer.[6]

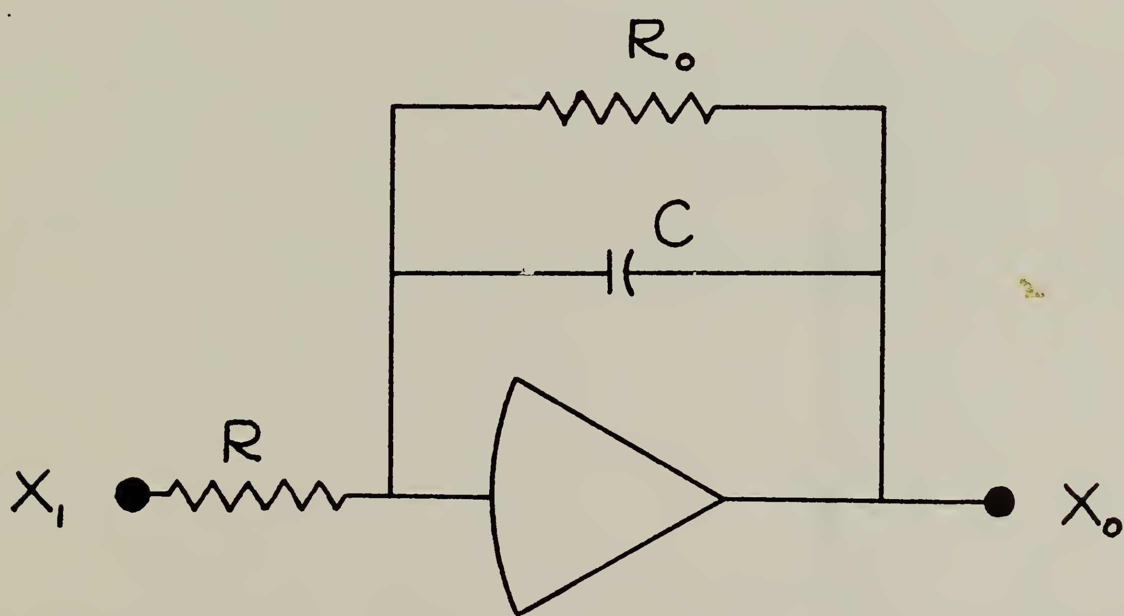
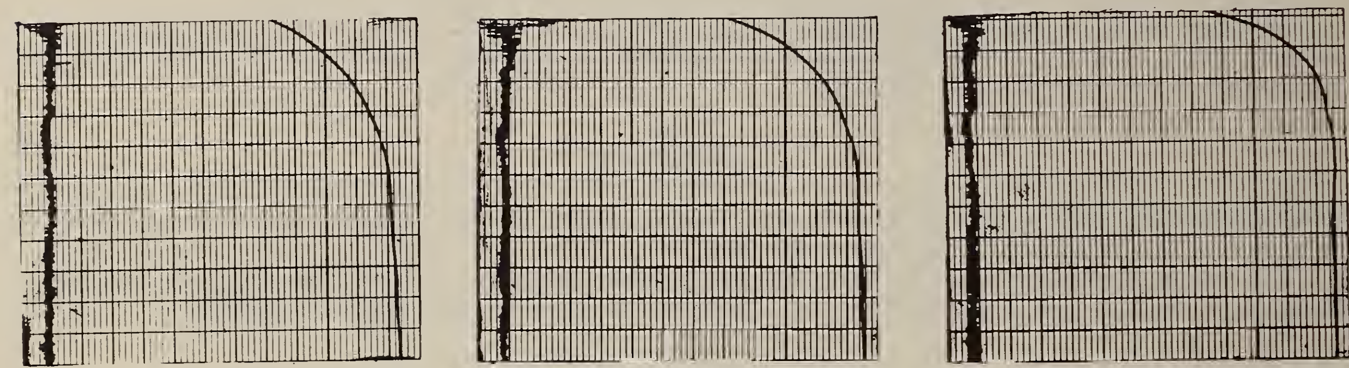


FIGURE 1.

[5] See p. 415 of footnote 2.

[6] The mention of trade products does not imply that they are recommended or endorsed by the Department of Agriculture over similar products of other companies not mentioned. Trade names are used here for convenience in reference only.

The value of the resistors, R and R_0, were set at 1 megohm each since these values were readily available within the operational amplifier. The value of the capacitor, C, was adjusted from 0.1 to 0.9 mfd to determine the optimum degree of filtering. The value of 0.5 mfd gave the best balance between the response and smoothing. As shown in figure 2, the values of 0.9 and 0.7 mfd for C were excessive since the filtered trace did not reach its maximum before the end of the test run, whereas the value of 0.5 mfd did reach a maximum soon after the start of the test run.



0.9 mfd. 0.7 mfd. 0.5 mfd.

FIGURE 2.—Filtered and unfiltered transducer output as obtained with various values of the capacitor, C. In each set of curves the filtered output is on the right and the unfiltered output is on the left.

U.S. GOVERNMENT PRINTING OFFICE : 1963 OL—692-684